1. A distant galaxy has a redshift \( z = (\lambda_{\text{observed}} - \lambda_{\text{emitted}})/\lambda_{\text{emitted}} \) of 0.2. According to Hubble’s law, how far away was the galaxy when the light was emitted if the Hubble constant is 72 (km/s)/Mpc?

2. A Cepheid variable star is observed with an apparent magnitude of 22 (see [http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry_magnitude.html#magnapparent](http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry_magnitude.html#magnapparent) for the notion of the magnitude of a star) and a period of 28 days. Using data from [http://hyperphysics.phy-astr.gsu.edu/hbase/astro/cepheid.html](http://hyperphysics.phy-astr.gsu.edu/hbase/astro/cepheid.html), determine the distance to this star.

3. Check by a direct calculation of the Christoffel symbols that the radial null rays in a FRW metric are geodesics, when suitably parameterised.

4. Verify that for FRW models with \( \rho + 3p \geq 0 \) and with non-positive cosmological constant the scale factor \( R \) is a concave function of \( t \) (i.e. \( \frac{d^2 R}{dt^2} \leq 0 \)). Assuming that \( R(t) \approx ct^\alpha \) as \( t \to 0 \), deduce from this that \( 1/H(t) \geq t \) for \( t > 0 \).

5. (For self-study) Check, or derive the \( k \neq 0 \) solutions of the Friedman equation in a non-empty matter-dominated universe with \( \Lambda = 0 \):

\[
\begin{align*}
  k = 1 : & \quad R = C(1 - \cos \eta) , \quad t = C(\eta - \sin \eta) , \\
  k = -1 : & \quad R = C(\cosh \eta - 1) , \quad t = C(-\eta + \sinh \eta) .
\end{align*}
\]

6. (For self-study) Suppose that the spatial volume of a closed, matter dominated, FRW universe with spherical space sections and vanishing cosmological constant is \( 10^{12} \text{Mpc}^3 \) at the moment of maximum expansion. What is the duration of this universe from big bang to big crunch in years?