Questions for the General Relativity II exam, July 2013

The candidate will draw randomly three questions from the list below, and will be asked to report on the topics drawn.

This list of questions gives a faithful representation of the contents of the lectures.

i. Lie bracket, Jacobi identity, Levi-Civita connection, Riemann curvature tensor and its properties, the geodesic deviation equation

ii. Inertial coordinates, Geodesic deviation (Jacobi equation), tidal forces

iii. Einstein equations and matter: examples of energy-momentum tensors, dust in general relativity, the continuity equation

iv. The Schwarzschild metric:the Eddington-Finkelstein extension; Time functions; the black hole; $r = 0$;

v. The Schwarzschild metric: Stationary observers, the Flamm paraboloid

vi. The Kruskal-Szekeres extension of the Schwarzschild metric

vii. The Schwarzschild metric: Conformal Carter-Penrose diagram

viii. The Schwarzschild metric: Geodesics, the interpretation of E, circular timelike geodesics, photons

ix. The Schwarzschild metric: Circular null geodesics, gravitational redshift, weak field light bending

x. The Schwarzschild metric: Massive test particles, Perihelion/periastron precession

xi. The parallel transport equation, geodetic precession along circular geodesics in Schwarzschild, Fermi-Walker transport

xii. Perfect fluids, general-relativistic Euler equations and their Newtonian limit, Newtonian thermodynamics interpretation

xiii. Linearized Einstein equations, TT-gauge, linearized waves

xiv. Slowly varying weak gravitational fields, quadrupole formula

xv. Spherically symmetric static stars: $g = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$,

$$G^0{}_0 = e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi\rho \ ,$$

$$G^1{}_1 = e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} = 8\pi p \ ,$$

derivation of the TOV equation

xvi. Spherically symmetric static stars, $g = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$,

$m' = 4\pi\rho r^2$, $e^{-\lambda(r)} = 1 - \dfrac{2m(r)}{r}$,

$p' = -\dfrac{(\rho + p)(4\pi p r^3 + m(r))}{r(r - 2m(r))}$

Newtonian limit, Buchdahl inequality, Chandrasekhar mass

xvii. FRW metrics: Hubble law, cosmological red-shift formula, the red shift-factor z and distance

xviii. FRW metrics:

$-G^0{}_0 = 3\dfrac{\dot{R}^2 + k}{R^2} = \kappa\rho + \Lambda$, $\kappa = 8\pi G/c^4$,

$G^i{}_i$ (no summation) $= -\dfrac{2\ddot{R}}{R} - \dfrac{\dot{R}^2 + k}{R^2} = \kappa p - \Lambda$,

radiation and matter solutions,

xix. The Friedman equation

$\dot{R}^2 = \dfrac{\kappa K}{3R^2} + \dfrac{\kappa E}{3R} + \dfrac{1}{3}\Lambda R^2 - k$,

its solutions, behavior for R small and large,

instability of static solutions