1 Show that $ds^\mu/d\tau = 0$ for a stationary observer in the Schwarzschild geometry carrying a gyroscope with spin vector s^μ.

2 Given a timelike unit vector field u, show that a linear plane wave solution $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ can be put in a gauge where $h^\alpha{}_\alpha = 0 = h_{\mu\nu}u^\nu$.

3 Let $g_{\mu\nu} = \eta_{\mu\nu} + A_{\mu\nu}\cos(k_\alpha x^\alpha)$ be the metric of a linearized plane wave in the gauge $A_{\mu\nu}k^\mu = 0 = A_{\mu\nu}U^\mu = A_\mu{}^\mu$, where $k^\mu\partial_\mu = \omega(\partial_t + \partial_z)$, $U^\mu\partial_\mu = \partial_t$. Using the variational principle for geodesics, write down the geodesic equations. Introducing the variables $u = t - z$, $v = t + z$, find the geodesics.

[*Hint*: You should find that there are two classes of geodesics. The first class have $u =$ constant, in which case you should be able to integrate the geodesic equations completely. For the geodesics with u non-constant, use u as a parameter along the geodesics, and solve the equations neglecting terms of order A^2.]

4 [*Problem for self-study, will not be covered in class*] Consider a timelike geodesic moving on a circular orbit in the $\theta = \pi/2$ plane in the Schwarzschild geometry with $r > 3m$.

a) Show that for every $r > 3m$ there exist timelike geodesics for which $\dot{r} = 0$.

b) Show that on such geodesics lying on the equatorial plane we have

$$\varphi = \varphi_0 + \Omega t\ , \quad \Omega = \frac{m^{1/2}}{r^{3/2}}\ , \qquad t = t_0 \pm \frac{Js}{\sqrt{mr}}\ ,$$

where $J = r^2 d\varphi/ds$, and s is the proper time along the geodesic.