1 Using the definition of the Lie derivative of the lectures, derive an explicit expression for the Lie derivative of a covector field, and show that the Lie derivative commutes with contractions.

2 [For self study, will not be covered in class] Show that

\[ [\mathcal{L}_X, \mathcal{L}_Y] = \mathcal{L}_{[X,Y]} \]

3 [For self study, will not be covered in class] Find the geodesics of the maximally symmetric Riemannian and Lorentzian manifolds. [Hint: it suffices to find a family with the property that all geodesics can be obtained from the members of the family by applying isometries.]

Using this, or otherwise, show that in the embedded model where the maximally symmetric manifold is a submanifold \( S_a \) of \( \mathbb{R}^3 \) as described in the lectures, the geodesics are intersections of \( S_a \) with planes through the origin.

4 A distant galaxy has a redshift \( z = (\lambda_{\text{observed}} - \lambda_{\text{emitted}})/\lambda_{\text{emitted}} \) of .2. According to Hubble's law, how far away was the galaxy when the light was emitted if the Hubble constant is 72 (km/s)/Mpc?

5 A Cepheid variable star is observed with an apparent magnitude of 22 (see http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry_magnitude.html#magnapparent for the notion of the magnitude of a star) and a period of 28 days. Using data from http://hyperphysics.phy-astr.gsu.edu/hbase/astro/cepheid.html, determine the distance to this star.

6 Show that all solutions of the linearisation of the Friedmann equation at \( \dot{R} = 0 \) are unstable.

7 Suppose that the spatial volume of a closed, matter dominated, FRW universe is \( 10^{12}\text{Mpc}^3 \) at the moment of maximum expansion. What is the duration of this universe from big bang to big crunch in years?

8 Recall that for FRW models with any combination of matter and radiation and with non-positive cosmological constant the scale factor \( R \) is a concave function of \( t \) (i.e. \( \frac{d^2 R}{dt^2} \leq 0 \)). Assuming that \( R(t) \sim t \) as \( t \to 0 \), deduce from this that \( 1/H(t) \geq t \) for \( t > 0 \).