1 Using the definition of the Lie derivative of the lectures, derive an explicit expression for the Lie derivative of a covector field, and show that the Lie derivative commutes with contractions.

2 [For self study, will not be covered in class] Show that

$$[\mathcal{L}_{\mathcal{X}}, \mathcal{L}_{\mathcal{Y}}] = \mathcal{L}_{[\mathcal{X},\mathcal{Y}]} \, .$$

3 [For self study, will not be covered in class] Find the geodesics of the maximally symmetric Riemannian and Lorentzian manifolds. *[Hint: it suffices to find a family with the property that all geodesics can be obtained from the members of the family by applying isometries.]*

Using this, or otherwise, show that in the embedded model where the maximally symmetric manifold is a submanifold $\mathcal{S}_a$ of $\mathbb{R}^3$ as described in the lectures, the geodesics are intersections of $\mathcal{S}_a$ with planes through the origin.

4 A distant galaxy has a redshift $z = (\lambda_{observed} - \lambda_{emitted})/\lambda_{emitted}$ of .2. According to Hubble's law, how far away was the galaxy when the light was emitted if the Hubble constant is 72 (km/s)/Mpc?

5 A Cepheid variable star is observed with an apparent magnitude of 22 (see `http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry_magnitude.html#magnapparent` for the notion of the magnitute of a star) and a period of 28 days. Using data from `http://hyperphysics.phy-astr.gsu.edu/hbase/astro/cepheid.html`, determine the distance to this star.

6 Show that all solutions of the linearisation of the Friedmann equation at $\dot{R} = 0$ are unstable.

7 Suppose that the spatial volume of a closed, matter dominated, FRW universe is $10^{12}\mathrm{Mpc}^3$ at the moment of maximum expansion. What is the duration of this universe from big bang to big crunch in years?

8 Recall that for FRW models with any combination of matter and radiation and with non-positive cosmological constant the scale factor R is a concave function of t (i.e. $\frac{d^2R}{dt^2} \leq 0$). Assuming that $R(t) \sim t$ as $t \to 0$, deduce from this that $1/H(t) \geq t$ for $t > 0$.