1 Let h be the metric on constant time slices in a spherically symmetric static star with constant density ρ. Show that h is the metric on a three sphere of a radius which you should determine. *[Hint: write the sphere* $S^3 \subset \mathbb{R}^4$ *as* $w = \sqrt{R^2 - x^2 - y^2 - z^2} = \sqrt{R^2 - r^2}$*, and calculate the metric induced on* S^3 *by the Euclidean metric* $dx^2 + dy^2 + dz^2 + dw^2 = dr^2 + r^2 d\Omega^2 + dw^2$*.]*

2 In the lecture we have estimated the Chandrasekhar mass by neglecting all coefficients of order one that occur in the calculation; we have also set $G = c = 1$. Do a proper calculation with all the coefficients in.

[Hint: As a first step, one needs to calculate the gravitational self-energy of a spherically symmetric Newtonian star with constant mass density ρ *within a ball o radius R. For this, one needs to find the Newton potential* ϕ*, solution of*

$$\Delta\phi = 4\pi G\rho \ ,$$

satisfying $\lim_{r\to\infty}\phi = 0$*. Here you can use the fact that the solution must be spherically symmetric (why?), so that*

$$\Delta\phi = \frac{1}{r^2}\partial_r(r^2\partial_r\phi) \ .$$

You can then use the fact that a shell at radius r of thickness dr will contribute

$$dU = 4\pi\rho r^2 \phi dr$$

to the total potential energy U of the star.]