Let $h$ be the metric on constant time slices in a spherically symmetric static star with constant density $\rho$. Show that $h$ is the metric on a three sphere of a radius which you should determine. [Hint: write the sphere $S^3 \subset \mathbb{R}^4$ as $w = \sqrt{R^2 - x^2 - y^2 - z^2} = \sqrt{R^2 - r^2}$, and calculate the metric induced on $S^3$ by the Euclidean metric $dx^2 + dy^2 + dz^2 + dw^2 = dr^2 + r^2 d\Omega^2 + dw^2$.]

In the lecture we have estimated the Chandrasekhar mass by neglecting all coefficients of order one that occur in the calculation; we have also set $G = c = 1$. Do a proper calculation with all the coefficients in.

[Hint: As a first step, one needs to calculate the gravitational self-energy of a spherically symmetric Newtonian star with constant mass density $\rho$ within a ball of radius $R$. For this, one needs to find the Newton potential $\phi$, solution of

$$\Delta \phi = 4\pi G \rho,$$

satisfying $\lim_{r \to \infty} \phi = 0$. Here you can use the fact that the solution must be spherically symmetric (why?), so that

$$\Delta \phi = \frac{1}{r^2} \partial_r (r^2 \partial_r \phi).$$

You can then use the fact that a shell at radius $r$ of thickness $dr$ will contribute

$$dU = 4\pi \rho r^2 \phi dr$$

to the total potential energy $U$ of the star.]