

Questions for the exam on General Relativity II, July 2012

The candidate will draw randomly two questions from the list below, and will be asked to report on the topics drawn.

This list of questions gives also a faithful representation of the contents of the lectures.

- i. Lie bracket, Jacobi identity, Levi-Civita connection, Riemann curvature tensor and its properties, the geodesic deviation equation
- ii. Inertial coordinates, Geodesic deviation (Jacobi equation), tidal forces
- iii. Einstein equations and matter: examples of energy-momentum tensors, dust in general relativity, the continuity equation
- iv. The Schwarzschild metric: the Eddington-Finkelstein extension; Time functions; the black hole; $r = 0$;
- v. The Schwarzschild metric: Stationary observers, the Flamm paraboloid
- vi. The Kruskal-Szekeres extension of the Schwarzschild metric
- vii. The Schwarzschild metric: Conformal Carter-Penrose diagram
- viii. The Schwarzschild metric: Geodesics, the interpretation of E , circular timelike geodesics, photons
- ix. The Schwarzschild metric: Circular null geodesics, gravitational redshift, weak field light bending
- x. The Schwarzschild metric: Massive test particles, Perihelion/periastron precession
- xi. The parallel transport equation, geodetic precession along circular geodesics in Schwarzschild
- xii. Fermi-Walker transport, Thomas precession, The Lens-Thirring effect
- xiii. Perfect fluids, general relativistic Euler equations and their Newtonian limit, Newtonian thermodynamics interpretation
- xiv. Integration on space-time, divergence theorem, conservation laws,
- xv. Linearized Einstein equations, TT-gauge, linearized waves
- xvi. Slowly varying weak gravitational fields, quadrupole formula

xvii. Spherically symmetric static stars: $g = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2$,

$$G^0_0 = e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) - \frac{1}{r^2} = -8\pi\rho ,$$

$$G^1_1 = e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right) - \frac{1}{r^2} = 8\pi p ,$$

derivation of the TOV equation

xviii. Spherically symmetric static stars, $g = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2$,

$$m' = 4\pi\rho r^2, e^{-\lambda(r)} = 1 - \frac{2m(r)}{r} ,$$

$$p' = - \frac{(\rho + p)(4\pi p r^3 + m(r))}{r(r - 2m(r))}$$

Newtonian limit, Buchdahl inequality, Chandrasekhar mass

xix. Transporting tensor fields, flows of vector fields

xx. The Lie derivative

xxi. Isometries, Killing vectors, maximally symmetric space-times

xxii. FRW metrics: cosmological red-shift formula, the red shift-factor z and distance

xxiii. FRW metrics:

$$-G^0_0 = 3 \frac{\dot{R}^2 + k}{R^2} = \kappa\rho + \Lambda, \kappa = 8\pi G/c^4,$$

$$G^i_i \text{ (no summation)} = -\frac{2\ddot{R}}{r} - \frac{\dot{R}^2 + k}{R^2} = \kappa p - \Lambda,$$

radiation and matter solutions,

xxiv. The Friedman equation

$$\dot{R}^2 = \frac{\kappa E}{3R^2} + \frac{\kappa E}{3R} + \frac{1}{3}\Lambda R^2 - k,$$

its solutions, behavior for R small and large,

instability of static solutions