1 Consider a timelike geodesic moving on a circular orbit in the $\theta = \pi/2$ plane in the Schwarzschild geometry with $r > 3m$. Show that

$$\varphi = \varphi_0 + \Omega t\ , \quad \Omega = \frac{m^{1/2}}{r^{3/2}}\ , \qquad t = t_0 \pm \frac{Js}{r}\ ,$$

where $J = r^2 d\varphi/ds$, and where $s$ is the proper time along the geodesic.

2 Show that $ds^\mu/d\tau = 0$ for a stationary observer in the Schwarzschild geometry carrying a gyroscope with spin vector $s^\mu$.

3 Consider the metric

$$g = g_{\mathrm{Schw}} - \frac{4J\sin^2\theta}{r} dt\, d\varphi\ ,$$

where $g_{\mathrm{Schw}}$ denotes the Schwarzschild metric.

i. Show that the world-line $\theta = 0$ is a geodesic in the metric $g$.

ii. Show that if $s^r = 0$ at some point then it is zero along the whole geodesic.

iii. Write-down explicitly the gyroscope equation on the last geodesic assuming that $s^r = 0$. (Can you solve it?)