1. Consider a timelike geodesic moving on a circular orbit in the $\theta = \pi/2$ plane in the Schwarzschild geometry with $r > 3m$. Show that

$$\varphi = \varphi_0 + \Omega t, \quad \Omega = \frac{m^{1/2}}{r^{3/2}}, \quad t = t_0 \pm \frac{Js}{r},$$

where $J = r^2d\varphi/ds$, and where $s$ is the proper time along the geodesic.

2. Show that $ds^\mu/d\tau = 0$ for a stationary observer in the Schwarzschild geometry carrying a gyroscope with spin vector $s^\mu$.

3. Consider the metric

$$g = g_{\text{Schw}} - \frac{4J \sin^2 \theta}{r} dt d\varphi,$$

where $g_{\text{Schw}}$ denotes the Schwarzschild metric.

   i. Show that the world-line $\theta = 0$ is a geodesic in the metric $g$.

   ii. Show that if $s^\mu = 0$ at some point then it is zero along the whole geodesic.

   iii. Write down explicitly the gyroscope equation on the last geodesic assuming that $s^\mu = 0$. (Can you solve it?)