1. Let \( g_{\mu\nu} = \eta_{\mu\nu} + A_{\mu\nu} \cos(k_\alpha x^\alpha) \) be the metric of a linearized plane wave in the gauge \( A_{\mu\nu} k^\mu = 0 = A_{\mu\nu} U^\mu = A_{\mu\mu} \). Using the variational principle for geodesics, write down the geodesic equations. Introducing the variables \( u = t - z, v = t + z \), find the geodesics.

[Hint: You should find that there are two classes of geodesics. The first class have \( u = \) constant, in which case you should be able to integrate the geodesic equations completely. For the geodesics with \( u \) non-constant, use \( u \) as a parameter along the geodesics, and solve the equations neglecting terms of order \( A^2 \).]

2. Consider a body of mass \( m_0 \) and angular momentum \( J_N \) moving in an elliptic, Kepler orbit of eccentricity \( e \) in the field of a central mass of mass \( m \). (See Problem Sheet 5, question 2.) Assume that, in Cartesian coordinates, this motion takes place in the \( z = 0 \) plane. Show that, at distance \( R \), and time \( t \), in the linearized approximation, the quadrupole moment of the system takes the form

\[
q_{ij} = 3m_0 r(\varphi(t))^2 \begin{pmatrix}
\cos^2 \varphi(t) & \sin \varphi(t) \cos \varphi(t) & 0 \\
\sin \varphi(t) \cos \varphi(t) & \sin^2 \varphi(t) & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

Using the explicit form of \( r(\varphi(t)) \) derived in Problem Sheet 5, question 2, derive an expression for the linearised metric perturbation \( \tilde{h}_{xx} \), to first order in the eccentricity \( e \).