Questions for the General Relativity II exam

The candidate will draw randomly two questions from the list below, and will be asked to report on the topic drawn.

This list of questions gives also a faithful representation of the contents of the lectures.

i. Lie bracket, Jacobi identity, Levi-Civita connection, Riemann curvature tensor and its properties, the geodesic deviation equation

ii. Inertial coordinates, Geodesic deviation (Jacobi equation), tidal forces

iii. Einstein equations and matter: examples of energy-momentum tensors, dust in general relativity, the continuity equation

iv. The Schwarzschild metric:the Eddington-Finkelstein extension; Time functions; the black hole; $r = 0$;

v. The Schwarzschild metric: Stationary observers, the Flamm paraboloid

vi. The Schwarzschild metric: Geodesics, the interpretation of E, circular timelike geodesics, photons

vii. The Schwarzschild metric: Circular null geodesics, gravitational redshift, weak field light bending

viii. The Schwarzschild metric: Massive test particles, Perihelion/periastron precession

ix. The parallel transport equation, geodetic precession along circular geodesics in Schwarzschild

x. Fermi-Walker transport, Thomas precession, The Lens-Thirring effect

xi. The Kruskal-Szekeres extension of the Schwarzschild metric

xii. The Schwarzschild metric: Conformal Carter-Penrose diagram

xiii. Perfect fluids, relativistic equation of motion, Newtonian thermodynamics interpretation

xiv. Integration on manifolds, divergence theorem, conservation laws,

xv. Linearized Einstein equations, TT-gauge, linearized waves

xvi. Slowly varying weak gravitational fields, quadrupole formula

xvii. Spherically symmetric static stars: $g = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$,

$G^0{}_0 = e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi\rho$,

$G^1{}_1 = e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} = 8\pi p$,

derivation of the TOV equation

xviii. Spherically symmetric static stars, $g = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$,

$m' = 4\pi\rho r^2$, $e^{-\lambda(r)} = 1 - \frac{2m(r)}{r}$,

$p' = -\frac{(\rho + p)(4\pi p r^3 + m(r))}{r(r - 2m(r))}$

Buchdahl inequality, Chandrasekhar mass

xix. The Lie derivative, an axiomatic approach, relation to isometries

xx. Transporting tensor fields, flows of vector fields

xxi. The Lie derivative: definition through flows

xxii. Isometries, Killing vectors, maximally symmetric space-times

xxiii. FRW metrics: cosmological red-shift formula, the red shift-factor z and distance

xxiv. FRW metrics:

$-G^0{}_0 = 3\frac{\dot{R}^2 + k}{R^2} = \kappa\rho + \Lambda$, $\kappa = 8\pi G/c^4$,

$G^i{}_i$ (no summation) $= -\frac{2\ddot{R}}{r} - \frac{\dot{R}^2 + k}{R^2} = \kappa p - \Lambda$,

radiation and matter solutions,

xxv. The Friedman equation

$\dot{R}^2 = \frac{\kappa E}{3R^2} + \frac{\kappa E}{3R} + \frac{1}{3}\Lambda R^2 - k$,

its solutions, behavior for R small and large,

instability of static solutions

xxvi. Variational principles: Euler-Lagrange equations, scalar fields

xxvii. The Hilbert variational principle

xxviii. The Palatini variational principle