You are expected to cross three problems amongst exercises 26, 27, 28 and 29. version revised after the classes in 2016; the question on energy threshold for production of pions is now in PS9 As usual, you are strongly encouraged to attempt more than three.

26 **Doppler effect I**

i. Consider an inertial frame attached to Earth. Prove that the world line of a spaceship with constant acceleration a passing through the origin at $\tau = 0$ with zero initial velocity is given by

$$(t(\tau), x(\tau)) = \left(a^{-1}\sinh(a\tau), a^{-1}(\cosh(a\tau) - 1)\right) , \tag{0.1}$$

where τ is the proper time of the spaceship. What is the asymptotic behaviour of the proper time $\tau \to \infty$ as a function of the time coordinate $t(\tau)$ of the spaceship?

ii. Express the four velocity of the spaceship in the inertial frame of Earth. Deduce the four velocity of the Earth in an inertial frame attached to the spacecraft.

iii. Consider a photon emitted from Earth towards the spaceship. Express the wave vector of the photon both in the instantaneous inertial frame of the spaceship and in the inertial frame of the earth. Deduce that the frequencies of the photon such as measured on the spaceship ω_S and on Earth ω_e are related by

$$\omega_e = \omega_S \sqrt{\frac{1 + v(\tau)}{1 - v(\tau)}},$$

where $v(\tau)$ is the velocity of the spacecraft in the earth frame.

iv. When, in Earth's time, and the astronaut's time, will the astronaut start seeing the blue oceans of the Earth ($\lambda_b = 450$) as red ($\lambda_r = 700nm$) if a equals the earth acceleration? How far will then the spaceship be?

v. Same question with $a = 5g$ (high-g rollercoaster)? $a = 9g$ (military pilot training)? $a = 100g$ (brief exposure in a crash)?

26 DF **Dopplereffekt I**:

Ein Raumschiff starte von der Erde und beschleunige mit konstanter (Eigen)beschleunigung $\alpha = g$. Wieviel Zeit vergeht für die Astronauten bis sie die blauen Ozeane ($\lambda_b = 450nm$) der Erde als rote ($\lambda_r = 700nm$) Ozeane sehen? Und in die Erde Zeit? Wie weit wird der Raumschiff gehen? (Es folgt, man müsste an Bord des Raumschiffs ein leistungsstarkes Teleskop haben.)

27 **Doppler effect II:** Let X be an observer and Y a source of photons. Let α be the angle between the direction of motion of the observer X and the photons (as observed from Y, the source of the photons.) Prove that there exists a unique angle α_{noD} of

the relative velocity v_{XY} of X and Y, so the Doppler effect disappears (that is to say that $\omega_X = \omega_Y$). Prove finally that, for small velocities, the following relation is true:

$$\alpha_{\text{noD}} = \frac{\pi}{2} - \frac{v_{XY}}{2} + O(v_{XY}^3)\,. \tag{0.2}$$

27 DF **Dopplereffekt II:** Sei α der Winkel zwischen der räumlichen Bewegungsrichtung des Beobachters X und des Photons (von Y aus gesehen, wo Y ist die Quelle des Photons). Zeige, dass es bei gegebener Relativgeschwindigkeit v_{XY} einen eindeutigen Winkel α_{noD} gibt, sodass keine Dopplerverschiebung eintritt (also $\omega_X = \omega_Y$). Beweise schliesslich, dass für kleine Geschwindigkeiten gilt

$$\alpha_{\text{noD}} = \frac{\pi}{2} - \frac{v_{XY}}{2} + O(v_{XY}^3)\,. \tag{0.3}$$

28 **Doppler effect III** A rigid ring of radius R=1 m spins with constant angular frequency $\omega = 2.1 * 10^8/s$ around its axis of symmetry. Every infinitesimal element of the ring emits electromagnetic radiation of length 450 nm (i.e. monochromatic blue light) as measured in the comoving frame of that element. What is the color of the ring perceived by i) an observer at the center of symmetry the ring, stationary with respect to that center, ii) by an observer situated somewhere on the ring, moving together with the ring? [*Hint: calculate the emitted and observed frequencies in the rest frame of the center of the ring.*]

29 **[Alice through the moving mirror – Doppler effect IV:]** A plane mirror moves in the direction of its normal with uniform velocity v, towards Alice, in Alice's rest frame S, and facing her (so in Alice's rest frame the velocity of the mirror is $(-v, 0, 0)$ with $v > 0$). A ray of light of frequency ω_1 strikes the mirror at an angle of incidence θ, and is reflected with frequency ω_2 at an angle of reflection φ. The purpose is to prove that

$$\frac{\tan\frac{1}{2}\theta}{\tan\frac{1}{2}\varphi} = \frac{c+v}{c-v}\,, \qquad \frac{\omega_2}{\omega_1} = \frac{\sin\theta}{\sin\varphi} = \frac{c\cos\theta + v}{c\cos\varphi - v} = \frac{c + v\cos\theta}{c - v\cos\varphi}\,. \tag{0.4}$$

Because of the geometry of the problem, we can assume that we are in three-dimensional Minkowski space-time.

i. Assuming that the wave vector of the incoming light ray with respect to the frame S of Alice is
$$k_1 = \omega_1(1, \cos\theta, \sin\theta)\,,$$
find the wave vector k_1' in the frame of the mirror.

ii. Assuming that the standard law of reflection in the mirror's frame holds, deduce that the wave vector of the outgoing light ray is, in the frame S':
$$k_2' = \omega_1(\gamma(1 + v\cos\theta), -\gamma(v + \cos\theta), \sin\theta)\,.$$

iii. Find k_2, by transforming back k_2' into the frame S.

iv. Introducing φ by $k_2 = \omega_2\,(1, -\cos\varphi, \sin\varphi)$, deduce that

$$\frac{\omega_2}{\omega_1} = \frac{\sin\theta}{\sin\varphi},\ \cos\varphi = \frac{(2v + v^2\cos\theta + \cos\theta)}{(1 + v^2 + 2v\cos\theta)},\ \sin\varphi = \frac{\sin\theta}{\gamma^2(1 + v^2 + 2v\cos\theta)}.$$

v. Use the formula

$$\tan\left(\frac{\varphi}{2}\right) = \frac{\sin\varphi}{1 + \cos\varphi}\ ,$$

to derive (0.4).