

- 1 Summation convention: For each of the following, either write out the equation with the summation signs included explicitly or explain why the equation is ambiguous or does not make sense. Provide a possible correct version, or versions, of the wrong or incoherent equations. (Recall that $\delta_b^a = 1$ if $a = b$ and is zero otherwise.)

$$\begin{array}{ll} \text{(i)} X^a = L_b^a M^{bc} \hat{X}_c & \text{(v)} X^a = L_b^a \hat{X}^b + M^{ab} \hat{X}^b \\ \text{(ii)} X^a = L_c^b M^c_d \hat{X}^d & \text{(vi)} X^a = L_b^a \hat{X}^b + M_c^a \hat{X}^c \\ \text{(iii)} \delta_b^a = \delta_c^a \delta_d^c & \text{(vii)} X^a = L_c^a \hat{X}^c + M_c^b \hat{X}^c \\ \text{(iv)} \delta_b^a = \delta_b^a \delta_c^c & \text{(viii)} X^a = L_c^a \hat{X}^c + \sum_c M^{ac} \hat{X}^c \end{array}$$

- 2 Sei $V = \mathbb{R}^4$ und $h^{\mu\nu} = h^{(\mu\nu)}$ von der Form

$$h^{\mu\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \mathbf{1} \end{pmatrix}$$

und $t_\mu = (1, \vec{0})$, wo $\mathbf{1}$ is die 3×3 Identität Matrix. Sei weiters

$$A^\mu_\nu = \begin{pmatrix} 1 & \vec{0} \\ -\vec{v} & \mathbf{1} \end{pmatrix}.$$

Beweise, dass

i.

$$A^\mu_\nu A^\lambda_\rho h^{\nu\rho} = h^{\mu\lambda}, \quad (1)$$

ii.

$$A^\mu_\nu t_\mu = t_\nu.$$

Was kann man sagen über alle A^μ_ν , die (1) erfüllen?

- 3 A beam of neutrinos is sent from CERN to the Gran Sasso National Laboratory for detection. The neutrinos travel with superluminal velocity $w > c$ measured at the rest frame of the Earth's crust, which we assume to be inertial. Another inertial observer travels in the same direction with subluminal velocity $v < c$. Assuming that the speed of light is c for all inertial observers, how fast does the moving observer have to travel in order to observe the detection in Gran Sasso happening *before* the beam is produced in CERN, i.e. to observe the neutrinos propagating backwards from Italy to Switzerland? Derive the general formula. What speed do you obtain if $w = (1 + 2 \times 10^{-5})c$? How does this value of w relate to the speed v of protons in the SPS and LHC in Geneva?

- 4 Schreibe die Lorentz-Transformation

$$\begin{aligned} t' &= \gamma \left(t - \frac{v}{c^2} x \right) \\ x' &= \gamma (x - vt) \end{aligned}$$

Übungen zur Vorlesung Relativitätstheorie und Kosmologie I: Problem Sheet 2

wobei

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2},$$

als (2×2) -Matrix $L(v)$. Beweise, dass

$$L(v_1) L(v_2) = L\left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)$$

Wie lautet die relativistische Addition für n gleiche Geschwindigkeiten? Wie oft muss man die Geschwindigkeit $c/2$ addieren, um die Geschwindigkeit $0,99c$, bzw $0,999c$ zu erreichen?