1 Sei $V = \mathbb{R}^4$ und $h^{\mu\nu} = h^{(\mu\nu)}$ von der Form

$$h^{\mu\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \mathbf{1} \end{pmatrix}$$

und $t_\mu = (1, \vec{0})$, wo $\mathbf{1}$ is die 3×3 Identität Matrix. Sei weiters

$$A^\mu{}_\nu = \begin{pmatrix} 1 & \vec{0} \\ -\vec{v} & \mathbf{1} \end{pmatrix}.$$

Beweise, dass

i.
$$A^\mu{}_\nu \, A^\lambda{}_\rho \, h^{\nu\rho} = h^{\mu\lambda}, \tag{1}$$

ii.
$$A^\mu{}_\nu \, t_\mu = t_\nu.$$

Was kann man sagen über alle $A^\mu{}_\nu$, die (1) erfüllen?

2 A beam of neutrinos is sent from CERN to the Gran Sasso National Laboratory for detection. The neutrinos travel with superluminal velocity $w > c$ measured at the rest frame of the Earth's crust, which we assume to be inertial. Another inertial observer travels in the same direction with subluminal velocity $v < c$. Assuming that the speed of light is c for all inertial observers, how fast does the moving observer have to travel in order to observe the detection in Gran Sasso happening *before* the beam is produced in CERN, i.e. to observe the neutrinos propagating backwards from Italy to Switzerland? Derive the general formula. What speed do you obtain if $v = (1 + 2 \times 10^{-5})c$? How does this compare to the speed of protons in the SPS and LHC in Geneva?

3 Schreibe die Lorentz–Transformation

$$\begin{aligned} t' &= \gamma \, (t - \frac{v}{c^2}\, x) \\ x' &= \gamma \, (x - v\, t) \end{aligned}$$

wobei

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2},$$

als (2×2)–Matrix $L(v)$. Beweise, dass

$$L(v_1)\, L(v_2) = L\left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)$$

Wie lautet die relativistische Addition für n gleiche Geschwindigkeiten? Wie oft muss man die Geschwindigkeit $c/2$ addieren, um die Geschwindigkeit 0,99c, bzw 0,999c zu erreichen?

4 Seien u^μ, v^ν Vektoren im Minkowski–Raum mit $u^\mu\, u_\mu = v^\mu\, v_\mu = -1$ und $u^\mu\, v_\mu < 0$. Hier man schreibt $u_\mu = \eta_{\mu\nu} u^\nu$, und $v_\mu = \eta_{\mu\nu} v^\nu$. Sei

$$L^\mu{}_\nu = \delta^\mu{}_\nu - 2 v^\mu\, u_\nu + (1 - u^\alpha\, v_\alpha)^{-1}\ (u^\mu + v^\mu)\,(u_\nu + v_\nu)\,.$$

Zeige, dass

i.

$$L^\mu{}_\nu\, u^\nu = v^\mu,$$

ii.

$$L^\mu{}_\nu\, L^\lambda{}_\rho\, \eta_{\mu\lambda} = \eta_{\nu\rho}.$$

Hint: calculations are simpler if you introduce $w^\mu := u^\mu + v^\mu$, $\phi := 1 - u^\alpha v_\alpha$, rewrite $L^\mu{}_\nu$ in terms of those, calculate $w^\mu u_\mu$, $w^\mu v_\mu$, deduce $w^\mu w_\mu$, and continue from there.