1. Las bracket over indices denote complete antisymmetrisation, and a parenthesis over indices denote complete symmetrisation: for example,
\[
A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu}), \quad A_{(\mu\nu)} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu}), \quad \delta_{\mu}^{[\alpha} \delta_{\rho}^{\gamma]} = \frac{1}{2}(\delta_{\mu}^{\alpha} \delta_{\rho}^{\gamma} - \delta_{\mu}^{\gamma} \delta_{\rho}^{\alpha}),
\]
eetc. Show that

i. \[
A_{[\mu\nu]} B_{\mu\nu} = A_{\mu\nu} B_{[\mu\nu]}, \quad A_{(\mu\nu)} B_{\mu\nu} = A_{\mu\nu} B_{(\mu\nu)},
\]

ii. \[
\delta_{\mu}^{[\alpha} \delta_{\rho}^{\gamma]} = \delta_{\mu}^{\alpha} \delta_{\rho}^{\gamma},
\]

iii. \[
\delta_{\mu}^{[\alpha} \delta_{\rho}^{\gamma]} = \delta_{\mu}^{\alpha} \delta_{\rho}^{\gamma},
\]

iv. \[
\epsilon^{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\rho} = -6 \delta_{\mu}^{\rho},
\]

v. \[
\epsilon^{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\rho} = -4 \delta_{\rho}^{\delta},
\]

2. Assuming the tensorial transformation law of \(F_{\mu\nu}\), derive the explicit formulae for the transformation laws of the electric and magnetic fields under a boost along the first coordinate axis.

3. 1) Assuming that \(\Lambda_{\alpha\beta}\) is a Lorentz matrix, show that \(\eta^{\alpha\beta} \Lambda_{\mu\beta} \eta_{\mu\nu}\) is inverse to \(\Lambda_{\alpha\beta}\).

2) Recall that we required that \(F_{\mu\nu}\) transforms as follows under Lorentz transformations: if \(\bar{x}^{\alpha} = \Lambda_{\alpha\beta} x^{\beta} + a^{\alpha}\), then
\[
\bar{F}^{\mu\nu}(\bar{x}) = \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} F^{\alpha\beta}(x),
\]
and that \(F_{\mu\nu}\) has been defined as
\[
F_{\mu\nu} := \eta_{\mu\alpha} \eta_{\nu\beta} F^{\alpha\beta}.
\]

Use 1) to show that

a) \(F_{\alpha\beta}\) transforms as
\[
\bar{F}_{\mu\nu}(\bar{x}) = (\Lambda^{-1})^{\mu}_{\alpha} (\Lambda^{-1})^{\nu}_{\beta} F_{\alpha\beta}(x)
\]
(this is called the transformation law of a two-covariant tensor);

b) \(F_{\alpha\beta} F^{\alpha\beta}\) and \(F^{\alpha\beta} F_{\alpha\beta}\) are invariant (more precisely, behave as scalars) under Lorentz transformations.

4. Express \(F_{\alpha\beta} F^{\alpha\beta}\) and \(F^{\alpha\beta} F_{\alpha\beta}\) in terms of \(\vec{E}\) and \(\vec{B}\).

5. Let \(\vec{E} \cdot \vec{B} = 0\), and suppose that \(|\vec{E}|^2 \neq |\vec{B}|^2\). Show that there exists a a Lorentz frame in which either \(\vec{E}\) or \(\vec{B}\) vanishes.