1 Zeige explizit, dass die Bedingung für Lorentztransformationen

$$L^{\mathrm{T}}\eta L = \eta$$

zehn unabhängige Gleichungen darstellt (woraus soll es folgen, dass die Lorentztransformationen eine 6-parametrige Gruppe bilden?).

2 Betrachte den Minkowski-Raum mit (pseudo)orthonormaler Basis $\{e_0, e_1, e_2, e_3\}$. Betrachte die Vektoren

$$v_0 = \begin{pmatrix} 1 \\ -1/2 \\ 0 \\ 0 \end{pmatrix}, \qquad v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix}.$$

Welcher dieser Vektoren ist zeit/licht/raumartig? Finde einen vierten Vektor, $v_3$, der (pseudo)orthogonal auf diese drei Vektoren steht. Ist $v_3$ zeit/licht/raumartig?

3 Seien $v$ und $w$ zeitartig und linear unabhängig. Zeige dass die Gerade $\{v+\lambda w \mid \lambda \in \mathbb{R}\}$ den Lichtkegel in zwei Punkten schneidet.

4 A clock $C$ is at rest at the spatial origin of an inertial frame $S$. A second clock $C'$ is at rest at the spatial origin of an inertial frame $S'$ moving with constant speed $v$ relative to $S$. The clocks read $t = t' = 0$ when the two spatial origins coincide. When $C'$ reads $t'_2$ it receives a radio signal from $C$ sent out when $C$ reads $t_1$. Draw a space-time diagram describing this process. Determine the space-time coordinates $(ct_2, x_2)$ in S of the point (event) at which $C'$ receives the radio signal. Hence show that

$$t_1 = t'_2 \sqrt{\frac{1 - v/c}{1 + v/c}}\ .$$

Is there a relationship with the Doppler effect?

5 The coordinates $(ct, x, y, z)$ and $(ct', x', y', z')$ in two inertial frames $S$ and $S'$ respectively are related by

$$t' = (\cosh\lambda)t - (\sinh\lambda)c^{-1}x \qquad (1a)$$

$$x' = -(\sinh\lambda)c\,t + (\cosh\lambda)x \qquad (1b)$$

$$y' = y \qquad (1c)$$

$$z' = z\,, \qquad (1d)$$

for a real number $\lambda$. Show that this defines a Lorentz transformation. If the origin in $S'$ has speed $V$ in $S$, what is $V$ in terms of $\lambda$? A particle has 3-velocity $(a, b, 0)$ as measured in $S$ and $(a', b', 0)$ as measured in S'. Find the relation between these 3-velocities in terms of $\lambda$. A light ray $\gamma$ in $S$ lies in the plane $z = 0$ and makes an angle $\alpha$ with the positive x-axis. Show that $\gamma$ lies in $z' = 0$ in $S'$. Show that, if $\gamma$ makes an angle $\alpha'$ with the positive $x'$-axis then $\tan(\alpha')/\tan(\alpha)$ is a function of $V$ and $\cos\alpha$, which should be found.