Exercises in Relativity and Cosmology II summer term 2014

Problem 25

Estimate

a) the order of magnitude of Ω_m on the surface of the earth,

b) the order of magnitude of Ω_{stat} for a low earth orbit.

Problem 26

Show that Fermi-Walker transport in Minkowski space implies the Thomas precession. *Hint*: The proof is short and does not require the calculation of the Thomas frequency.

Problem 27

Conclude from 26. that spin precession in Minkowski space is described by the transport equation

$$\frac{d\vec{s}}{dt} = \gamma^2 \vec{v}(\frac{d\vec{v}}{dt} \cdot \vec{s})$$

and derive from the rotational part of the r.h.s. of this equation the nonrelativistic limit of the Thomas frequency

$$\vec{\Omega}_{Th} \approx -\frac{1}{2}\vec{v} \times \frac{d\vec{v}}{dt}.$$

Optional: How would one derive the exact Thomas frequency from the transport equation?

Problem 28

Let M_1 be the real line $\mathbb{R}$ with the standard differentiability structure and M_2 the differentiable manifold defined by $\mathbb{R}$ with the atlas $\{x \mapsto x^3\}$. Show:

a) The differentiability structures of M_1 and M_2 are not compatible.

b) The identity on $\mathbb{R}$ is not a diffeomorphism from M_1 to M_2.

c) Give an example of a diffeomorphism from M_1 to M_2.

Problem 29

Show that the abstract definition of a tangent vector v implies that $v(c) = 0$ for a constant function c.

Problem 30

Show that the tangent vectors $\partial_i|_P$ form a basis for T_P. Use and prove the following lemma: An arbitrary function $\phi \in C^\infty(\mathbb{R}^n)$ can be written as $\phi(x) = \phi(0) + g_i(x)x^i$ with $g_i(0) = \partial_i\phi|_0$.

Problem 31

Show abstractly that the Lie bracket of two vector fields is a vector field and compute the components of $[u, v]$.