

## Übungen zur Vorlesung Relativitätstheorie und Kosmologie II: Problem Sheet 11

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- 1 A distant galaxy has a redshift  $z = (\lambda_{observed} - \lambda_{emitted})/\lambda_{emitted}$  of .2. According to Hubble's law, how far away was the galaxy when the light was emitted if the Hubble constant is 72 (km/s)/Mpc?
- 2 A Cepheid variable star is observed with an apparent magnitude of 22 (see [http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry\\_magnitude.html#magnapparent](http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry_magnitude.html#magnapparent) for the notion of the magnitude of a star) and a period of 28 days. Using data from <http://hyperphysics.phy-astr.gsu.edu/hbase/astro/ceheid.html>, determine the distance to this star.
- 3 Check by a direct calculation of the Christoffel symbols that the radial null rays in a FRW metric are geodesics, when suitably parameterised.
- 4 Verify that for FRW models with  $\rho + 3p \geq 0$  and with non-positive cosmological constant the scale factor  $R$  is a concave function of  $t$  (i.e.  $\frac{d^2R}{dt^2} \leq 0$ ). Assuming that  $R(t) \approx ct^\alpha$  as  $t \rightarrow 0$ , deduce from this that  $1/H(t) \geq t$  for  $t > 0$ .
- 5 (For self-study) Check, or derive the  $k \neq 0$  solutions of the Friedman equation in a non-empty matter-dominated universe with  $\Lambda = 0$ :

$$k = 1 : \quad R = C(1 - \cos \eta) , \quad t = C(\eta - \sin \eta) , \quad (1)$$

$$k = -1 : \quad R = C(\cosh \eta - 1) , \quad t = C(-\eta + \sinh \eta) . \quad (2)$$

- 6 (For self-study) Suppose that the spatial volume of a closed, matter dominated, FRW universe with spherical space sections and vanishing cosmological constant is  $10^{12}\text{Mpc}^3$  at the moment of maximum expansion. What is the duration of this universe from big bang to big crunch in years?