1 A distant galaxy has a redshift $z = (\lambda_{observed} - \lambda_{emitted})/\lambda_{emitted}$ of .2. According to Hubble's law, how far away was the galaxy when the light was emitted if the Hubble constant is 72 (km/s)/Mpc?

2 A Cepheid variable star is observed with an apparent magnitude of 22 (see `http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry_magnitude.html#magnapparent` for the notion of the magnitute of a star) and a period of 28 days. Using data from `http://hyperphysics.phy-astr.gsu.edu/hbase/astro/cepheid.html`, determine the distance to this star.

3 Recall from *PS* 11 that, given a one-form α, we defined its exterior derivative $d\alpha$ to be the two-form with components

$$(d\alpha)_{ab} = \partial_a \alpha_b - \partial_b \alpha_a.$$

Consider the manifold $M = \mathbb{R}^{2n}$, with coordinates (x^i, p_i), $i = 1, \ldots, n$. (We will also denote the coordinates by q^a, $a = 1, \ldots, 2n$, with $q^1 = x^1, \ldots, q^n = x^n, q^{n+1} = p_1, \ldots, q^{2n} = p_n$). Let

$$\alpha = \sum_{i=1}^{n} p_i dx^i.$$

(So the components of α are $\alpha_{x^i} = p_i, \alpha_{p_i} = 0$.)

Calculate $\omega := -d\alpha$. Check that the matrix ω_{ab} is invertible, find its inverse. $((\mathbb{R}^{2n}, \omega)$ is an example of what is called a *symplectic manifold*.)

Let $H = H(x, p)$ be a function on M, hence its differential dH is

$$dH = \frac{\partial H}{\partial q^a} dq^a = \sum_{i=1}^{n} \left[\frac{\partial H}{\partial x^i} dx^i + \frac{\partial H}{\partial p_i} dp_i \right].$$

The *Hamiltonian vector field* corresponding to H is the vector field, X_H, on M defined by the equations

$$\omega_{ab} X_H^a = \frac{\partial H}{\partial q^b}, \qquad a, b = 1, \ldots, 2n.$$

Check that, in index-free notation, this is the same as $\omega(X_H, \cdot) = dH$.

Calculate the components of the Hamiltonian vector field X_H. Write down the equations for an integral curve of the vector field X_H. Do you recognise these equations?

Given two functions f and g, calculate $[X_f, X_g]$. What do you obtain if $f = p_i$ and $g = x^j$? Do you recognize the resulting operation on functions?

4 Let X be a Killing vector on a compact Riemannian manifold with non-positive Ricci tensor: $R_{ij} X^i X^j \leq 0$. Integrating by parts in the integral

$$\int X^i X^j{}_{;ij}$$

show that X is covariantly constant, and that the isometry group of compact Riemannian Einstein (this means that $R_{ij} = \frac{R}{n} g_{ij}$) manifolds with $R < 0$ is discrete. *[Hint: Use the equation satisfied by the second derivatives of a Killing vector. Also note that "integration by parts" is equivalent to the fact that on a compact manifold without boundary we have* $\int Y^j{}_{;j} = 0$ *for any differentiable vector field* Y*.]*