You are expected to cross three problems amongst exercises 42, 43, 44 and 45. As usual, you are strongly encouraged to attempt more than three.

We write $\det g_{\mu\nu}$ for the determinant of a matrix $(g_{\mu\nu})$, $g^{\mu\nu}$ for the matrix inverse to $g_{\alpha\beta}$ (thus, $g^{\mu\nu}g_{\nu\alpha} = \delta^\mu_\alpha$), and $\phi^A{}_{,\mu}$ for $\partial\phi^A/\partial x^\mu$.

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•*0.1:* **ptc:** *the field theory questions w[…] 2016 after the problem sheet was po[…] therefore this PS has never been pos[…] this PS should come before the Eins[…] things and indices and so for in the […]*

42 Let $A_{\mu\nu}$ be an invertible matrix, with inverse $B^{\mu\nu}$. Show that

$$\frac{\partial(\det A_{\mu\nu})}{\partial A_{\rho\sigma}} = (\det A_{\mu\nu})B^{\sigma\rho} . \tag{1}$$

Deduce that

$$\partial_\alpha \sqrt{\det A_{\mu\nu}} = \frac{1}{2}\sqrt{\det A_{\mu\nu}} B^{\rho\sigma}\partial_\alpha A_{\sigma\rho} . \tag{2}$$

43 Write-down the Euler-Lagrange equations, the canonical energy-momentum tensor

$$t^\mu{}_\nu = \frac{\partial \mathscr{L}}{\partial \phi^A{}_{,\mu}}\phi^A{}_{,\nu} - \mathscr{L}\delta^\mu_\nu ,$$

and the symmetric energy-momentum tensor

$$T_{\mu\nu} = 2\frac{\partial \mathscr{L}}{\partial g^{\mu\nu}} ,$$

for the following Lagrageans:

i. $\mathscr{L} = \frac{1}{2}\left(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - m^2\phi^2\right)\sqrt{-\det g_{\mu\nu}}$
(massive scalar field);

ii. $\mathscr{L} = \left(\frac{1}{8\pi}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu}\right)\sqrt{-\det g_{\mu\nu}}$,
where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (Maxwell field); and

iii. $\mathscr{L} = \frac{1}{8\pi}\left(e^{-a\phi}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu} + g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)\sqrt{-\det g_{\mu\nu}}$,
where as before $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $a \in \mathbb{R}$ (dilaton-Maxwell system).

In the calculation of the Euler-Lagrange equations and of the canonical energy-momentum tensor you can assume that $g_{\mu\nu} = \eta_{\mu\nu}$, the Minkowski metric.

44 a) Show that a photon cannot spontaneously disintegrate into an electron-positron pair.

b) Recall that the Zero Momentum (ZM) frame is defined as the inertial frame in which the space-momentum vector of the system vanishes. Find the velocity of the ZM frame of two photons of frequencies ν_1 and ν_2 that travel in the positive and negative x-directions respectively.

45 Radiation energy from the sun is received on earth on the equator at the rate of 1.94 calories per minute per square centimeter. Assuming the distance of the sun to be 150 000 000 km, find the total mass lost by the sun per second, and the force exerted by solar radiation on a black disk of the same diameter as the earth (use 12 800 km), at the location of the earth.

46 How fast must a particle move before its *kinetic energy*, defined as the difference between the total energy and the rest energy, equals the rest energy?

47 Consider the photoproduction of pions

$$\gamma + p \to \pi^0 + p \ ,$$

with the target proton at rest. What is the minimal energy of the photon γ for this process to take place? (Assume that the mass of the proton is 0.94 Gev, and the mass of the pion 140 Mev.) Compare the resulting frequency to that of hard X rays ($\lambda_X = 10^{-2} nm$). What will the result be if the target proton is moving in the same direction as the photon?

48 **Energy efficiency of the production of particles** We consider the production of pions by collision of protons in a particle accelerator. The effective energy E_{out} of the production of particles is the sum of the energies of the particles $\sum_j m_j$ (up to a c^2-factor in International Units) produced by the collision. The minimal kinetic energy, in the rest frame of the accelerator, necessary to produce the new particles from the collision of the original particles is denoted by E_{in}. The coefficient of energy efficiency of the particle generation is defined as $\kappa = \frac{E_{\text{out}}}{E_{\text{in}}}$. Compute κ for

$$\mathsf{p} + \mathsf{p} \to \mathsf{p} + \mathsf{p} + \pi^0 \ ,$$

where p is a proton and π^0 is a pion when

a) one proton is at rest in the rest frame of the accelerator;

b) both protons are colliding with opposite velocities as measured in the rest frame of the accelerator.

c) Does κ depend on the frame in which it is computed?