You are expected to prepare solutions and cross in the "Kreuzerl-Liste" (via moodle) to three problems out of the list below.

9 Let $\{e_0, e_1, e_2, e_3\}$ be a (pseudo)-orhonormal basis of Minkowski spacetime. Consider the vectors:

$$v_0 = \begin{pmatrix} 1 \\ -1/2 \\ 0 \\ 0 \end{pmatrix}, \qquad v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix}.$$

Which of these vectors is space/time/lightlike? Find a fourth vector v_3, othogonal to these three vectors. Is v_3 space/time/lightlike?

9, DF Sei $\{e_0, e_1, e_2, e_3\}$ eine (pseudo-)orthonormale Basis des Minkowski-Raums. Betrachten Sie die Vektoren

$$v_0 = \begin{pmatrix} 1 \\ -1/2 \\ 0 \\ 0 \end{pmatrix}, \qquad v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix}.$$

Welcher dieser Vektoren ist zeit-/licht-/raumartig? Finden Sie einen vierten Vektor, v_3, der (pseudo-)orthogonal auf diesen drei Vektoren steht. Ist v_3 zeit-/licht-/raumartig?

10 A clock C is at rest at the spatial origin of an inertial frame S. A second clock C' is at rest at the spatial origin of an inertial frame S' moving with constant speed v relative to S. The clocks read $t = t' = 0$ when the two spatial origins coincide. When C' reads t'_2 it receives a radio signal from C sent out when C reads t_1. Draw a space-time diagram describing this process. Determine the space-time coordinates (ct_2, x_2) in S of the point (event) at which C' receives the radio signal. Hence show that

$$t_1 = t'_2 \sqrt{\frac{1 - v/c}{1 + v/c}}\,.$$

Is there a relationship with the Doppler effect?

11 Let u^μ, v^ν be two vectors of the Minkowski space satisfying $u^\mu u_\mu = v^\mu v_\mu = -1$ and $u^\mu v_\mu < 0$. We remind the convention $u_\mu = \eta_{\mu\nu} u^\nu$, and $v_\mu = \eta_{\mu\nu} v^\nu$. Consider the linear mapping:

$$L^\mu{}_\nu = \delta^\mu{}_\nu - 2v^\mu u_\nu + (1 - u^\alpha v_\alpha)^{-1} \, (u^\mu + v^\mu)(u_\nu + v_\nu)\,. \qquad (0.1)$$

Prove that

i. $L^\mu{}_\nu u^\nu = v^\mu$,

ii. $L^{\mu}{}_{\nu}\, L^{\lambda}{}_{\rho}\, \eta_{\mu\lambda} = \eta_{\nu\rho}$.

[Hint: calculations are simpler if you introduce $w^{\mu} := u^{\mu} + v^{\mu}$, $\phi := 1 - u^{\alpha} v_{\alpha}$, rewrite $L^{\mu}{}_{\nu}$ in terms of those, calculate $w^{\mu} u_{\mu}$, $w^{\mu} v_{\mu}$, deduce $w^{\mu} w_{\mu}$, and continue from there.]

What is the interpretation of $L^{\mu}{}_{\nu}$?

Do matrices of the form (0.1) form a group?

11, DF Seien u^{μ}, v^{ν} Vektoren im Minkowski–Raum mit $u^{\mu}\, u_{\mu} = v^{\mu}\, v_{\mu} = -1$ und $u^{\mu}\, v_{\mu} < 0$. (Hier ist die Konvention $u_{\mu} := \eta_{\mu\nu} u^{\nu}$, $v_{\mu} := \eta_{\mu\nu} v^{\nu}$ verwendet worden.) Sei

$$L^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - 2 v^{\mu}\, u_{\nu} + (1 - u^{\alpha}\, v_{\alpha})^{-1}\, (u^{\mu} + v^{\mu})\, (u_{\nu} + v_{\nu})\,.$$

Zeigen Sie, dass

i. $L^{\mu}{}_{\nu}\, u^{\nu} = v^{\mu}$,

ii. $L^{\mu}{}_{\nu}\, L^{\lambda}{}_{\rho}\, \eta_{\mu\lambda} = \eta_{\nu\rho}$.

Was ist die Interpretation von $L^{\mu}{}_{\nu}$?

12 Find four linearly independent a) spacelike vectors, b) timelike vectors, and c) null vectors.

13 Let v and w be two timelike and linearly independent vectors. Prove that the line $\{v + \lambda w \,|\, \lambda \in \mathbb{R}\}$ intersects the light cone at the origin in exactly two points.

13, DF Seien v und w zwei zeitartige, linear unabhängige Vektoren. Zeigen Sie, dass die Gerade $\{v + \lambda w \,|\, \lambda \in \mathbb{R}\}$ den Lichtkegel mit Vertex am Ursprung in zwei Punkten schneidet.

15 [For self-study, will not be covered in class.] Prove explicitly that the property of the Lorentz transformations

$$L^{\mathrm{T}} \eta L = \eta$$

forms a system of ten independant equations in the coefficients of L (from which it should follow that the group of Lorentz transformation is a 6-parameter group?). •0.1

•0.1: **ptc:** *add something about the ex antisymmetric matrix*

15, DF [Zum Selbststudium.] Zeigen Sie explizit, dass die Bedingung für Lorentztransformationen

$$L^{\mathrm{T}} \eta L = \eta$$

zehn unabhängige Gleichungen darstellt (woraus folgt dann, dass die Lorentztransformationen eine 6-parametrige Gruppe bilden?).