8. In an inertial frame S, an electromagnetic field is determined by the 4-vector potential A defined by

$$A = (\phi, \mathbf{0}).$$

A particle having charge e and rest-mass m moves in this electromagnetic field with 4-velocity V, where

$$V = \gamma(\mathbf{v})(c, \mathbf{v})$$

and

$$\gamma(\mathbf{v}) = (1 - \frac{\mathbf{v}\cdot\mathbf{v}}{c^2})^{-\frac{1}{2}}.$$

Show that the Lorentz force law for the particle reduces to the equations

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma(\mathbf{v})\mathbf{v}) = \quad \frac{e}{m} \ \nabla\phi,$$

where t is the time co-ordinate in S, and

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \ e\mathbf{v}\cdot\nabla\phi,$$

where E is the energy of the particle, as measured in S.

Now consider the case in which, at a point $(t, \mathbf{r})$ in S,

$$- \ \phi(t, \mathbf{r}) = \frac{e'}{r},$$

where $r = (\mathbf{r}\cdot\mathbf{r})^{\frac{1}{2}}$ and $ee' < 0$. Show that the Lorentz force law for the particle is satisfied by a motion in which the particle moves in a circle, centred at the spatial origin, with constant angular 3-velocity. In this case show that the speed v of the particle satisfies the quartic equation

$$v^4 + \lambda^2(v^2 - c^2) = 0,$$

where

$$\lambda = \frac{ee'}{mca},$$

a being the radius of the circle.