

8. In an inertial frame S , an electromagnetic field is determined by the 4-vector potential A defined by

$$A = (\phi, \mathbf{0}).$$

A particle having charge e and rest-mass m moves in this electromagnetic field with 4-velocity V , where

$$V = \gamma(\mathbf{v})(c, \mathbf{v})$$

and

$$\gamma(\mathbf{v}) = \left(1 - \frac{\mathbf{v} \cdot \mathbf{v}}{c^2}\right)^{-\frac{1}{2}}.$$

Show that the Lorentz force law for the particle reduces to the equations

$$\frac{d}{dt}(\gamma(\mathbf{v})\mathbf{v}) = -\frac{e}{m}\nabla\phi,$$

where t is the time co-ordinate in S , and

$$\frac{dE}{dt} = -e\mathbf{v} \cdot \nabla\phi,$$

where E is the energy of the particle, as measured in S .

Now consider the case in which, at a point (t, \mathbf{r}) in S ,

$$\phi(t, \mathbf{r}) = \frac{e'}{r},$$

where $r = (\mathbf{r} \cdot \mathbf{r})^{\frac{1}{2}}$ and $ee' < 0$. Show that the Lorentz force law for the particle is satisfied by a motion in which the particle moves in a circle, centred at the spatial origin, with constant angular 3-velocity. In this case show that the speed v of the particle satisfies the quartic equation

$$v^4 + \lambda^2(v^2 - c^2) = 0,$$

where

$$\lambda = \frac{ee'}{mca},$$

a being the radius of the circle.